Transboundary Pollution, Tax Competition, and the
Efficiency of Uncoordinated Environmental Regulation*

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Abstract

Under capital tax competition, surprisingly, Ogawa and Wildasin (2009, American Economic Review, 99(4), 1206-1217) find that uncoordinated policymaking leads to a first-best outcome even in the presence of transboundary pollution. However, I show that if the level of environmental regulation is endogenized, the regulation level becomes too loose compared with the optimum (“race to the bottom”). Thus, despite the efficiency result of Ogawa and Wildasin (2009), efforts to achieve international environmental policy coordination are needed. I then examine the dependence of this result on the level of decisive voter’s capital endowment. The regulation is inefficiently loose in many cases, but it can be too strict if the decisive voter’s capital endowment is above the average. Thus, the possibility of “race to the top” cannot be eliminated. The inefficiency result does not generally depend on the timing of policymaking, although the efficiency may be restored in the limit case where the decisive voter has no capital at all.

Keywords: uncoordinated environmental regulation, transboundary pollution, race to the bottom, race to the top, tax competition

JEL classification: D62, H23, H71, H87, Q58

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1 Introduction

A lot of serious environmental issues, such as global warming, air pollution, and acid rain, are characterized by transboundary pollution. Since transboundary pollution creates a negative externality, it seems difficult to achieve an efficient outcome without a centralized authority or an international agreement.\(^1\) Realizing this, countries have made many efforts to achieve international coordination on environmental issues with transboundary pollution, such as Convention on Long-range Transboundary Air Pollution and Kyoto Protocol.

However, according to Ogawa and Wildasin (2009), these efforts may not be necessary if capital is mobile and jurisdictions engage in capital tax competition (Zodrow and Mieszkowski, 1986; Wilson, 1986). Ogawa and Wildasin (2009) show that, surprisingly, if jurisdictions engage in capital tax competition, decentralized policymaking leads to an efficient outcome even in the presence of transboundary pollution. Moreover, their result holds even when jurisdictions have heterogeneous preferences or production technologies. Empirical evidence shows that tax competition may take place both at the local and at the national government level.\(^2\) Thus, the strikingly powerful efficiency result of Ogawa and Wildasin (2009) implies that efforts to reach an agreement on global environmental issues may not be necessary if jurisdictions compete for mobile capital.

In this article, however, I show that the efficiency result of Ogawa and Wildasin (2009) crucially depends on an implicit assumption that the level of environmental regulation is exogenous. This assumption is not fully satisfactory because governments in the real world seem to have a discretion over the level of environmental regulation and they can use it as an instrument to attract capital, as pointed out in Oates and Schwab (1988) and a lot of subsequent works. This drives jurisdictions to compete not only in tax rates but also in the level of environmental regulation. By endogenizing the level of environmental regulation, I show that the level of uncoordinated environmental regulation becomes too loose compared with the optimum, that is, “race to the bottom” takes place. Ogawa and Wildasin (2009) implicitly assume that jurisdictions compete for mobile capital only via taxes. Such mode of intergovernmental competition works so that the transboundary pollution is internalized. However, intergovernmental competition via both taxes and environmental standards does not work that way. This is an important qualification to the efficiency result of Ogawa and Wildasin (2009). This result implies that, as opposed to the implication of Ogawa and Wildasin (2009), it is necessary to try to reach an international environmental agreement. Without it, regulation becomes too loose and an efficient outcome cannot be reached.

Ogawa and Wildasin (2009) assume that jurisdictions maximize the average utility of residents. However, governments’ behavior in the real world is often affected by political considerations. For

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\(^1\)An empirical literature shows that transboundary pollution leads to an inefficient outcome if environmental policymaking is decentralized. See Sigman (2002, 2005) and Banzhaf and Chupp (2012) among others.

\(^2\)See, for example, Hayashi and Boadway (2001) and Devereux et al. (2008).
example, it may behave according to the interest of the median voter or particular political parties. Recognizing this, there is growing literature which introduces political economic factors into tax competition models and analyzes how they alter the results (e.g., Borck, 2003; Sato, 2003; Lai 2014; Ogawa and Susa, 2017). Following this literature, I analyze how the efficiency of uncoordinated regulation is affected if governments do not maximize the average utility but the utility of the decisive voter.

I consider heterogeneity in capital endowment within a jurisdiction and see how my main result depends on the amount of capital the decisive voter holds. Given today’s significant inequality in wealth both before and after redistribution (Piketty, 2014; Looney and Moore, 2016), the preference heterogeneity arising from inequality in capital seems important in considering capital taxation despite that redistributive policies are financed by other revenue sources as well. When capital endowment of the decisive voter is less than the average, uncoordinated regulation results in an inefficiently loose level. However, when it is more than the average and the utility function is non-linear, uncoordinated regulation may become inefficiently strict ("race to the top"). This result is surprising because "race to the top" may occur even in the presence of transboundary pollution.

I also consider the situation in which tax rates are chosen after regulation levels are determined. Such a situation is reasonable if implementing a new environmental regulation is relatively time-consuming. In this case, governments take into account that changing regulation levels may induce changes in the equilibrium tax rates, which is not considered in previous literature such as Oates and Schwab (1988). Even in this case, the presence of transboundary pollution makes the equilibrium regulation inefficiently loose, which confirms the robustness of my main result. However, I also show that the efficiency is restored in the limit case where the decisive voter has no capital at all. This is a knife-edge result in the sense that it does not hold if the decisive voter has strictly positive amount of capital. Still, it would be a useful benchmark in assessing the scope and the limitation of Ogawa and Wildasin’s (2009) efficiency result.

This paper can be seen as reconciling two strands of literature. The first strand of literature is concerned about the efficiency of uncoordinated environmental regulation under intergovernmental competition for capital. Oates and Schwab (1988) is a pioneering work in this area. They suppose

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3 In this paper, I consider source-based taxes on capital like the corporate income taxes and property taxes. In OECD countries, on average, these two taxes collect around 5% of GDP every year (OECD, 2017). Although the figure is smaller than personal income taxes (around 8% of GDP) and taxes on goods and services (around 10% of GDP), source-based taxes seem to be non-negligible revenue sources of redistributive policies.

4 Withagen and Halsema (2013) also introduce transboundary pollution into a tax competition model. They point out that uncoordinated tax rates may become higher than coordinated ones owing to environmental concerns. Although this paper also points out the possibility of too strict uncoordinated environmental policies, my primary objective is to analyze the efficiency of the equilibrium regulation. On the other hand, Withagen and Halsema (2013) consider the regulation level to be exogenous and so do not address this issue.

5 Almost all previous studies focus on the “horizontal” competition. Thus, the “vertical” relationship between a central authority and local governments has been generally ignored in the context of environmental policy. An exception is Böhringer et al. (2016), who explicitly include vertical fiscal externalities and consider their implications on environmental policies.
that governments can choose capital tax rates and environmental regulation levels, both of which affect capital allocations. They show that the level of uncoordinated environmental regulation is efficient and capital is not taxed at all. However, they also show that the level of decentralized regulation is inefficiently loose in many cases. On the other hand, Wellisch (1995) and Glazer (1999) show that decentralized environmental regulation may be too severe if firms are mobile and a class of policy instrument is used. These works do not allow for transboundary pollution, although it is a key factor in environmental issues. This is presumably because, until Ogawa and Wildasin (2009), a simple and general framework which introduces transboundary pollution into capital tax competition did not exist.

The second strand of literature, initiated by Ogawa and Wildasin (2009), analyzes the efficiency of uncoordinated policymaking under tax competition in the presence of interjurisdictional spillovers. Ogawa and Wildasin (2009) show that, surprisingly, the allocation of resources is efficient under fairly general conditions even in the presence of environmental spillovers. There are several works which give qualifications to the efficiency result of Ogawa and Wildasin (2009). Eichner and Runkel (2012) show that the efficiency result of Ogawa and Wildasin (2009) no longer holds if capital supply is elastic. Fell and Kaffine (2014) show that the efficiency result of Ogawa and Wildasin (2009) may not hold if capital retirement or public abatement activity is incorporated so that the total amount of pollution is endogenous. Eichner and Runkel (2014) consider two production sectors: the dirty sector and the clean sector. Again, the efficiency result of Ogawa and Wildasin (2009) does not hold because capital allocation between the dirty sector and the clean sector is distorted.

Table 1 summarizes the results of existing studies and what the current paper is going to show. The row of Table 1 represents whether there is transboundary pollution and the column represents which policy instruments are used in intergovernmental competition for mobile capital. Ogawa and Wildasin (2009) show that intergovernmental competition for capital makes uncoordinated policymaking efficient even in the presence of transboundary pollution. However, in contrast to Oates and Schwab (1988), regulation levels are given exogenously and so they cannot be used in the competition. I extend Ogawa and Wildasin’s (2009) framework by endogenizing the level of environmental regulation as in Oates and Schwab (1988) and characterize the efficiency of uncoordinated environmental regulation. Because transboundary pollution is an important feature of many serious

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7 Ogawa and Wildasin (2009) build on Zodrow and Mieszczynski (1986), which is the standard model of capital tax competition. There are some works which include both tax competition and interjurisdictional spillovers in a different setting. For example, Cremer and Galvani (2004) consider the effect of economic integration and tax harmonization in the presence of tax competition and transboundary pollution. Withagen and Halsema (2013) construct a tax competition model in which tax competition may lead to a strict environmental policy. Davies and Naughton (2014) consider emission tax competition in the presence of transboundary pollution and analyze whether proximity matters in achieving international cooperation.
8 Ogawa and Wildasin (2009) is also extended to analyze other important environmental issues. For example, Eichner and Pethig (2014) use the model of Eichner and Runkel (2012), which is an extension of Ogawa and Wildasin (2009), to analyze self-enforcing environmental agreements under tax competition.
environmental issues, such characterization seems indispensable. This attempt corresponds to filling in the bottom-right box of Table 1. I show that uncoordinated regulation becomes inefficient. It is too loose if there is transboundary pollution, but political economic factors may make the regulation too strict.

Recently, Fell and Kaffine (2014) introduce abatement activities into Ogawa and Wildasin (2009) and show that the level of abatement activities becomes inefficiently low. In their model, the level of abatement activities are independent of capital market and thus choices about abatement activities cannot be used in the intergovernmental competition for capital. This is presumably because Fell and Kaffine’s (2014) main focus is endogenizing the total pollution and abatement activities are introduced as a simple way for achieving that purpose. In contrast, as in Oates and Schwab (1988), my model introduces environmental regulation so that it can be used in attracting capital. Thus, each jurisdiction’s environmental policymaking is linked not only by transboundary pollution but also by the capital market, which makes the underlying mechanism quite different from that of Fell and Kaffine (2014).  

This difference partly comes from the fact that the regulation cost is paid by firms in my framework, while the cost of abatement activities is incurred by residents in Fell and Kaffine (2014). In footnote 13, I discuss why my assumption is reasonable in analyzing environmental regulation.

This paper is organized as follows. Section 2 introduces the model. The coordinated (first-best) regulation level is derived in Section 3. Section 4 analyzes the efficiency of uncoordinated regulation under intergovernmental competition for capital. Section 5 considers an alternative timing assumption in which tax rates are chosen after regulation levels are determined. I discuss some extensions in Section 6. Section 7 concludes.

### 2 The Model

I introduce the model, which is a variant of that used by Ogawa and Wildasin (2009). While making some simplifications, I also extend it so that each jurisdiction chooses its environmental regulation

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levels and the capital endowment within a jurisdiction is heterogeneous.

There are two symmetric jurisdictions \((i = 1, 2)\). Each jurisdiction’s population \(L\) is normalized to 1. This economy uses two production factors, capital \(K\) and labor \(L\), to produce the numeraire good. Capital is assumed to be freely mobile while labor is immobile. The production technology is described by a homogeneous of degree one production function \(F(K, L) = Lf(k) = f(k)\) with \(f' > 0, f'' < 0,\) and \(f''' > 0\).\(^{10}\) To ensure positive production in each jurisdiction, I assume \(f(0) = 0, f'(0) = \infty,\) and \(f'(\infty) = 0\). I also assume \(f''\) and \(f'''\) are finite for any positive finite value of \(k\).

Within each jurisdiction \(i\), there are perfectly competitive markets for capital and labor. They are, thus, priced at their marginal productivity, \(r_i = f'(k_i)\) and \(w_i = f(k_i) - k_i f'(k_i)\) where \(k_i, r_i,\) and \(w_i\) are the capital in jurisdiction \(i\), the gross return to capital in jurisdiction \(i\), and the gross wage rate in country \(i\), respectively.

Capital supply is fixed at the global level:

\[
k_1 + k_2 = 2\overline{k} > 0.
\]

The initial endowment of capital in each economy is \(\overline{k}\). I assume that capital is owned by local residents as in Ogawa and Wildasin (2009). However, in the spirit of Borek (2003) and Ogawa and Susa (2017), I extend their assumption on capital ownership by allowing for heterogeneous capital endowment of residents within a jurisdiction.\(^{11}\) Individual \(m\) in a jurisdiction has \(\theta_m \overline{k}\) unit of capital, where \(\overline{k}\) is the average amount of capital in a jurisdiction. If individual \(m\)’s capital endowment is more than the average, \(\theta_m > 1\), and vice versa. If \(\theta_m = 1\), individual \(m\) has the average amount of capital. Let \(\theta_D \in [0, \infty)\) be the capital endowment of the decisive voter, whose utility the jurisdiction tries to maximize.\(^{12}\) If \(\theta_D = 1\), each jurisdiction maximizes the average utility of the residents, which is standard in tax competition literature including Zodrow and Mieszkowski (1986) and Ogawa and Wildasin (2009).

The environmental damage jurisdiction \(i\) receives is denoted by \(e_i\). It is given by

\[
e_i = \alpha_i k_i + \beta \alpha_j k_j
\]

where \(\beta \in [0, 1]\) describes the degree of transboundary pollution. Greenhouse gases, for example, may correspond to \(\beta = 1\) since the level of global warming would be determined by the total

\(^{10}\)\(f''' > 0\) is a sufficient condition to avoid ambiguity in the comparative statics in Section 5. Other results do not utilize this assumption. Commonly used production functions such as \(f(k) = k^m\) with \(m \in (0, 1)\) satisfy \(f''' > 0\).

\(^{11}\)The efficiency result of Ogawa and Wildasin (2009) remains true even if each government maximizes the utility of the decisive voter because they assume that jurisdictions take the net return to capital as given. In the current paper, jurisdictions do not take it as given. However, it can be shown that if regulation levels are exogenously given and the same across jurisdictions as in Ogawa and Wildasin (2009), the equilibrium is efficient. Thus, the assumption of governments maximizing the decisive voter’s utility per se does not invalidate the efficiency result of Ogawa and Wildasin (2009).

\(^{12}\)I do not specify how the decisive voter is selected. For example, it may represent the preference of the median voter. It may be a reduced form expression of the preference of the party which obtains political power through rent-seeking activities (Sato, 2003; Lai, 2014).
emission of greenhouse gases in the world. Acid rain and air pollution may correspond to the case of \( \beta \in (0, 1) \) since the pollutants do not become entirely mixed up in the air. When \( \beta = 0 \), there is no environmental spillover as in Oates and Schwab (1988). \( \alpha_i \) represents the level of environmental regulation of jurisdiction \( i \). Even when jurisdiction \( i \) and \( j \neq i \) have the same amount of capital, a jurisdiction with lower \( \alpha \) emits less pollutants. Thus, small \( \alpha_i \) corresponds to strict environmental regulation since it means the environmental damage per capital is small. Note that when \( \alpha_1 = \alpha_2 = \alpha \) where \( \alpha \) is the exogenously given regulation level, the environmental damage reduces to that in Ogawa and Wildasin (2009). The environmental damage given by (2) is a generalization of what they have.

Jurisdiction \( i \) decides on two policy instruments; capital tax \( t_i \) and environmental regulation \( \alpha_i \). The environmental regulation entails a cost \( C(\alpha_i)k_i \) where \( C(\alpha) \) is twice differentiable, \( C' < 0, C'' > 0 \), \( \lim_{\alpha \to 0} C'(\alpha) \to -\infty \), and \( \lim_{\alpha \to \infty} C'(\alpha) \to 0 \). The cost is paid in the numeraire good. Function \( C(\alpha) \) determines cost of the environmental regulation per capital. This cost is incurred by firms.\(^{13}\)

Capital is freely mobile and hence the net return to capital is equalized across jurisdictions. Denoting the world net return to capital by \( \rho \), I have

\[
f'(k_1) - t_1 - C(\alpha_1) = f'(k_2) - t_2 - C(\alpha_2) = \rho. \tag{3}
\]

(1) and (3) determine the capital allocations and the net return to capital \( \rho \) given \( (t_1, t_2, \alpha_1, \alpha_2) \). I assume \( f' \) is large enough so that \( \rho \geq 0 \) holds and (1) holds with equality in all relevant cases.\(^{14}\)

Jurisdictions are large in the sense that they take into account terms of trade effects.\(^{15}\) That is, each jurisdiction recognizes that its behavior affects \( \rho \). As (3) implies, capital allocations depend both on the capital tax rates and the level of the environmental regulation. Each jurisdiction recognizes it in choosing policy variables. From (1) and (3),

\[
\begin{align*}
\frac{\partial k_i}{\partial t_i} &= \frac{1}{f''(k_1) + f''(k_2)} < 0, \\
\frac{\partial k_j}{\partial t_j} &= \frac{-1}{f''(k_1) + f''(k_2)} > 0, \\
\frac{\partial \rho}{\partial t_i} &= \frac{-f''(k_j)}{f''(k_1) + f''(k_2)} < 0
\end{align*} \tag{4}
\]

\(^{13}\)Let \( \rho_i \) denote the net return to capital in jurisdiction \( i \). Then, the representative firm in jurisdiction \( i \) chooses \( k_i \) to maximize the profit \( f(k_i) - \rho_i k_i - t_i k_i - C(\alpha_i)k_i - w_i \), taking \( (\rho_i, \alpha_i, t_i, w_i) \) as given. The first order condition implies \( \rho_i = f'(k_i) - t_i - C(\alpha_i) \) and \( \rho_1 = \rho_2 = \rho \) must hold due to the free mobility of capital, which lead to (3). I assume that the regulation cost is paid by firms for two reasons. Firstly, it seems to describe an aspect of environmental regulation in the real world. Many governments set an emission standard which firms are required to follow. Indeed, such regulation is widely observed all over the world (e.g., Clean Air Act in the US, the Industrial Emissions Directive in the EU, and Air Pollution Control Act in Japan). It effectively requires firms to pay the regulation cost. Secondly, most of previous works which analyze the efficiency of environmental regulation under tax competition (e.g., Oates and Schwab, 1988; Wellisch, 1995) also assume that regulation cost is incurred by firms. Thus, adopting this assumption makes it easier to compare these works with the current paper.

\(^{14}\)Fell and Kaffine (2014) show that when the net return to capital \( \rho \) becomes zero and some capital is not invested, the efficiency result of Ogawa and Wildasin (2009) no longer holds. In this paper, I do not consider this possibility.\(^{15}\)

\(^{15}\)I argue in Section 6 that the main proposition (Proposition 1) can be shown to be valid even if there are many jurisdictions and the terms of trade effects are negligible. It should be noted that the assumption of large jurisdictions itself does not break the efficiency result of Ogawa and Wildasin (2009) because, as shown by Eichner and Runkel (2012), terms of trade effects vanish at the symmetric equilibrium.
and

\[
\frac{\partial k_i}{\partial \alpha_i} = \frac{C'(\alpha_i)}{f''(k_1) + f''(k_2)} > 0, \quad \frac{\partial k_i}{\partial \alpha_j} = -\frac{C'(\alpha_j)}{f''(k_1) + f''(k_2)} < 0, \quad \frac{\partial p}{\partial \alpha_i} = -\frac{f''(k_j)C'(\alpha_i)}{f''(k_1) + f''(k_2)} > 0 \quad (5)
\]

for \(i = 1, 2\) and \(j \neq i\). (4) and (5) show that a jurisdiction can attract capital by two measures: lowering the capital tax rate or relaxing the environmental regulation. The capital-attracting effect of loosening regulation is absent in Ogawa and Wildasin (2009). This effect is considered in Oates and Schwab (1988), although they do not consider transboundary pollution. In this sense, the current model can be seen as combining these two works. (4) and (5) also show the effect of policy variables on the net return to capital. The net return to capital becomes high when tax rates are low and the regulation is lax. When the regulation is loose, firms incur only small cost. This makes capital more attractive.

Each government acts as if it maximizes the utility of an individual with \(\theta^D\) (the decisive voter).\(^{16}\) Following Fell and Kaffine (2014), I assume that the tax revenue is paid back to residents.\(^{17}\) Then, the amount of consumption of the individual with \(\theta^D\), \(x_i^D\), is given by the sum of the wage, the capital earnings, and the transfer from the government. The utility of the individual with \(\theta_D\) is given by

\[
U_i^D = x_i^D - e_i = f(k_i) - k_if'(k_i) + t_ik_i + \theta^D \rho \bar{k} - \alpha_i k_i - \beta \alpha_j (2\bar{k} - k_i). \quad (6)
\]

That is, the utility is assumed to be linear.\(^{18}\) It should be noted that, since I have not specified the magnitude of \(\alpha\), the analysis remains valid regardless of the extent of environmental damage.\(^{19}\) Note also that regulation cost enters (6) through the net return to capital \(\rho\) defined by (3). Thus, although regulation cost is paid by firms, the burden is eventually on the capital owners.\(^{20}\)

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\(^{16}\)Here, I implicitly impose two assumptions. First, governments do not care about factors other than the utility of the decisive voter. Second, the distinguishing characteristic of voters is only their capital endowment. Although these assumptions may indeed be restrictive in some cases, the analysis in this paper remains valid even when these assumptions are relaxed in some important ways. For example, the government of jurisdiction \(i\) may care about the social welfare of jurisdiction \(j (SW_j)\) in addition to the utility of the decisive voter. As explained in Section 3, maximizing \(SW_i\) is equivalent to maximizing the utility of the voter with \(\theta_m = 1\). Thus, if the government has an objective function of the form \(\gamma SW_i + (1 - \gamma)U_i^D\) where \(\gamma \in [0, 1]\), the analysis is exactly the same as the case where the parameter of the decisive voter is modified as \(\gamma + (1 - \gamma)\theta^D\). I also briefly discuss the implication of the heterogeneity in preferences for environmental quality at the end of Section 4.

\(^{17}\)Eichner and Runkel (2012) show that the result of Ogawa and Wildasin (2009) remains valid if both public goods and labor taxation are excluded. In other words, public goods do not play a crucial role in their analysis. Note also that this specification implicitly assumes that when the tax rate is negative, the deficit is financed by the uniform lump-sum taxes on residents.

\(^{18}\)Extension to the general utility function \(U(x_i, e_i)\) is extensively discussed in Section 4. Note that, although restrictive, linear utility assumption is often adopted in tax competition literature to obtain clear-cut results. See, for example, Peralta and Ypersele (2005), Haufler and Lülfesmann (2015), Ogawa et al. (2016), and Ogawa and Susa (2017).

\(^{19}\)More formally, the analysis remains the same even if I alternatively assume \(U_i^D = x_i^D - \omega e_i\) where \(\omega \in (0, \infty)\) is a scaling parameter.

\(^{20}\)In Fell and Kaffine (2014), abatement cost is financed by lump-sum taxes from residents. The current framework has two important features. First, changes in the level of regulation induces capital reallocation which affect \(k_i, k_j\), and \(\rho\). The decisive voter takes into account changes in these endogenous variables through adjustments in the capital market. This feature is similar to Oates and Schwab (1988), which represent the cost of strict regulation by the
As a benchmark, I assume that jurisdiction \( i (= 1; 2) \) independently and simultaneously chooses the tax rates \( (t_i) \) and the environmental regulation level \( (\alpha_i) \). This situation is reasonable when a government cannot credibly commit to policy variables. It is also plausible when changing tax rates and regulation levels are equally time-consuming. However, other timing assumptions may be more plausible in other contexts. For example, to adjust to a new regulation, a firm may have to introduce some new facilities to purify polluted air or water. Sometimes it may have to invent a new technology to follow the new regulation. As a result, implementing new regulation takes a lot of time. Changing tax rates, on the other hand, does not require such adjustments. Therefore, a jurisdiction may have to commit to certain regulation level for a relatively long time. In Section 5, I discuss how my results may or may not be modified if regulation levels should be chosen before choosing tax rates. In Section 6, I also consider the alternative assumption that tax rates are chosen before choosing regulation levels.

3 Coordinated (First-Best) Regulation

As a benchmark, I characterize the efficiency of coordinated environmental regulation. I follow Eichner and Runkel (2014) in formulating the social planner’s problem. I assume that the social planner chooses \( (1; 2; t_1; t_2) \) to maximize the total utility

\[
[x_1 - e_1] + [x_2 - e_2] = \left( f(k_1) - k_1 f'(k_1) + t_1 k_1 + \rho k_1 \right) - \left[ \alpha_i k_i + \beta \alpha_j k_j \right] + \left( f(k_j) - k_j f'(k_j) + t_j k_j + \rho k_j \right) - \left[ \alpha_j k_j + \beta \alpha_i k_i \right]
\]

subject to (1) and (3).\(^{21}\) Two remarks are in order. First, since the population of each jurisdiction is normalized to 1 and the utility is linear, the average utility within jurisdiction \( i \) (i.e., the utility of a resident with \( \theta_m = 1 \)) is written in exactly same way as the total welfare of jurisdiction \( i \). Second, the social planner in this model is constrained by the mobility of capital, implying that the social planner also takes (4) and (5) into account. The solution to this problem describes the policies when a centralized authority, such as EU and United Nations, can freely manipulate each jurisdiction’s policy variables but it cannot restrict the movement of capital. I assume this situation to highlight the role of policy coordination among jurisdictions. However, as noted by Eichner and Runkel (2014), it can be verified that the coordinated regulation coincides with the first-best (Pareto-optimal) solution chosen by the social planner unconstrained by the mobility of capital.\(^{22}\)

decreased marginal productivity. Second, when \( \theta^D < (>)1 \), the decisive voter pays less (more) than the regulation cost per resident at the symmetric equilibrium.

\(^{21}\)There is no reason for the social planner to focus on the utility of the decisive voter when each person’s utility equally matters for her. Thus, it seems more natural to assume that she cares about the total (average) utility of jurisdictions.

\(^{22}\)The first-best optimum is given as the solution to the following social planner’s problem: the social planner chooses \( k_i, x_i, \) and \( \alpha_i \) \((i = 1, 2)\) to maximize \( x_1 - e_1 \) subject to (i) \( x_2 - e_2 = \bar{u} \), (ii) \( f(k_1) + f(k_2) - x_1 - x_2 - C(\alpha_1)k_1 - C(\alpha_2)k_2 = 0 \), and (iii) \( k_1 + k_2 = 2\bar{k} \), where \( \bar{u} \) is the utility level of jurisdiction 2 and exogenously given. Note that in this case, the
Using (1) to eliminate \( k_j \) and taking derivatives, the first order condition for \( \alpha_i \) is given by

\[
\frac{\partial k_i}{\partial \alpha_i} (-k_i f''(k_i) + t_i - \alpha_i + \beta \alpha_j + k_j f''(k_j) - t_j + \alpha_j - \beta \alpha_i) + 2 \frac{\partial \rho}{\partial \alpha_i} k - k_i - \beta k_i = 0,
\]

where \( i = 1, 2, j \neq i \), and \( \frac{\partial \rho}{\partial \alpha_i} = -\frac{f''(k_i)C(\alpha_i)}{f''(k_1) + f''(k_2)} \). Since jurisdictions are symmetric, I focus on the symmetric solution. Evaluating the first order conditions for \( \alpha_i \) at \( t_i = t_j = t^o \), \( \alpha_i = \alpha_j = \alpha^o \), and \( k_i = k_j = k \), (8) is rewritten as

\[
-C'(\alpha^o) = 1 + \beta. \tag{9}
\]

Note that \( \alpha^o \) does not depend on tax rates. That is, as long as each jurisdiction sets the same tax rate, the efficient regulation is characterized by (9). I call \( \alpha^o \) “efficient regulation.” The LHS of (9) is the (social) marginal cost of strengthening the environmental standard. On the other hand, the RHS is the marginal social benefit from tightening regulation. The efficient \( \alpha^o \) is thus attained when the social marginal cost is equal to the social marginal benefit.

### 4 Uncoordinated Regulation under Intergovernmental Competition for Capital

Now, let me solve the model and derive the equilibrium regulation level. Each jurisdiction chooses both tax rates and regulation levels independently and simultaneously. For \( i = 1, 2 \) and \( j \neq i \), the FOC for \( \alpha_i \) is given by

\[
\frac{\partial U_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial \rho} \frac{\partial \rho}{\partial \alpha_i} = -k_i + \frac{C'(\alpha_i)}{f''(k_i) + f''(k_j)} \left[ -k_i f''(k_i) + t_i - \alpha_i + \beta \alpha_j - \theta D k f''(k_j) \right] = 0 \tag{10}
\]

and the FOC for \( t_i \) is

\[
\frac{\partial U_i}{\partial t_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial t_i} + \frac{\partial U_i}{\partial \rho} \frac{\partial \rho}{\partial t_i} = k_i + \frac{1}{f''(k_i) + f''(k_j)} \left[ -k_i f''(k_i) + t_i - \alpha_i + \beta \alpha_j - \theta D k f''(k_j) \right] = 0 \tag{12}
\]

social planner is not restricted by the mobility of capital. The solution must satisfy \( k_1 = k_2 \) and \( \alpha_1 = \alpha_2 \). To see this, suppose \((k_1, k_2, \alpha_1, \alpha_2, x_1, x_2)\) is a solution to the above problem. By eliminating \( x_1 \) and \( x_2 \), the problem is essentially maximizing \( f(k_1) + f(k_2) - (c_1 + c_2) - (C(\alpha_1)k_1 + C(\alpha_2)k_2) - \pi \) subject to \( k_1 + k_2 = 2k \). Suppose I change regulation levels in the following way: marginally change \( \alpha_1 \) by \( da_1 \) and \( \alpha_2 \) by \( da_2 = -\frac{k_1}{k_2} da_1 \). This keeps the total environmental damage \( c_1 + c_2 \) unchanged, while the total regulation cost \( C(\alpha_1)k_1 + C(\alpha_2)k_2 \) changes by \( [C'(\alpha_1) - C'(\alpha_2)]k_1 da_1 \), which must be zero at the optimum. Since \( C' \) is strictly increasing, \( \alpha_1 = \alpha_2 \) must hold. Given that the regulation level is the same at the solution, the capital allocation does not affect the total environmental damage. Thus, \( k_1 = k_2 \) must also hold at the solution to achieve productive efficiency \((f'(k_1) = f'(k_2))\). At the symmetric solution, (9) readily follows from the first order conditions.

\(^{23}\)At the symmetric solution, the specific value of the tax rate \( t^o \) is indeterminate. One unit increase in \( t^o \) raises the total reimbursement of tax revenues by \( 2\theta k \) and consumption increases by the same amount. At the same time, it decreases \( \rho \) by one unit, which reduces consumption by \( 2\theta k \) in total. Thus, the specific value of \( t^o \) does not matter for the total welfare.
Since all jurisdictions are identical, in the following analysis, I focus on the symmetric equilibrium with $\alpha_1 = \alpha_2 = \alpha^c$ and $t_1 = t_2 = t^c$ where the superscript $c$ denotes equilibrium values under intergovernmental competition. Note that, at the symmetric equilibrium, $k_1 = k_2 = \bar{k}$ from (3).

Rearranging (11) by using (13), at the symmetric equilibrium, I obtain

$$-C'(\alpha^c) = 1.$$  
(14)

By Comparing of (9) and (14), I obtain the following main result:

**Proposition 1**

Regardless of the value of $D$, the uncoordinated environmental regulation under intergovernmental competition $\alpha^c$ is inefficiently loose ($\alpha^c > \alpha^o$) as long as there is transboundary pollution ($\beta \in (0, 1]$). It is efficient ($\alpha^c = \alpha^o$) if there is no transboundary pollution ($\beta = 0$).

**Proof:** Note that $-C'$ is strictly decreasing, (9) and (14) imply that $\alpha^c \geq \alpha^o$ where the equality holds if and only if $\beta = 0$. □

Proposition 1 states that if there is no transboundary pollution, the uncoordinated regulation level becomes efficient. This result is reminiscent of Oates and Schwab (1988), who show that in the absence of transboundary pollution, the uncoordinated regulation level is efficient under capital tax competition.

However, this efficiency result no longer holds if transboundary pollution is present ($\beta \in (0, 1]$). Regardless of the severity of transboundary pollution, the regulation level remains the same. Thus, it is inefficiently loose in the presence of transboundary pollution. As seen in (11), strengthening regulation directly benefits citizens by reducing pollution by $k_i$. An increase in tax rates, on the other hand, directly benefits citizens because it increases the tax revenue by $k_i$. Thus, the direct effect of these two policy changes is the same. Loosening regulation and increasing tax rates also induce capital reallocations and change the net return to capital, which indirectly affect citizens’ utility. However, these indirect effects should be equalized since the two direct effects are the same in magnitude and the total marginal benefit is zero in the equilibrium. It requires that the marginal outflow of capital by strengthening regulation and increasing the tax rate should be the same. From (4) and (5), it ensures that a jurisdiction internalizes the marginal benefit of reducing the pollution for itself. However, the marginal benefit for other jurisdiction is not internalized. As a result, Proposition 1 follows.

This result is an important qualification to Ogawa and Wildasin (2009). They show that under fairly general conditions, intergovernmental competition for capital makes decentralized policymaking efficient even in the presence of transboundary pollution. According to this, it seems that international agreements about global environmental issues are not necessary. However, Ogawa and Wildasin (2009) implicitly assume that environmental damage per capital is exogenous. Thus, jurisdictions compete only in taxes. In reality, governments have a discretion in choosing the level of
environmental regulation and many international environmental agreements are closely related to the regulation. Realizing this, Oates and Schwab (1988) and the subsequent works suppose that jurisdictions can attract capital by loosening environmental regulation. Proposition 1 indicates that when jurisdictions compete for capital both in taxes and regulations, intergovernmental competition does not lead to an efficient outcome in the presence of transboundary pollution.\textsuperscript{24}

In the remaining of this section, I analyze how the results are modified when the utility function is not linear. In Section 2, I introduce the model in which residents have linear utilities $U(x_i, e_i) = x_i - e_i$. As noted in Section 2, this specification is less restrictive than it appears at first sight because any severity of environmental damages is allowed for. Still, it is worthwhile to investigate whether my results carry over to the case of general utility function $U(x_i, e_i)$, where $U_x > 0$ and $U_e < 0$. I first confirm the robustness of Proposition 1. Then, I demonstrate that decisive voter’s capital endowment matters when utility function is non-linear. In particular, I show that the equilibrium regulation may be too strict when the decisive voter has more than the average amount of capital.

An important issue in the analysis with the general utility function is that defining the social planner’s problem becomes much more delicate, especially because of the difficulty in specifying the distribution of capital endowment and the form of the social welfare function. As in the case of linear utility, I assume that the social planner regards jurisdiction $i$’s welfare as the utility of the voter with $\theta_m = 1$ ($U_i|_{\theta_m=1}$) and she maximizes $U_1|_{\theta_m=1} + U_2|_{\theta_m=1}$. Note that welfare implications should be interpreted with caution because the utility of the voter with $\theta_m = 1$ can no longer be interpreted as the total utility of residents. Nevertheless, it is useful in facilitating the comparison with results under the assumption of linearity.\textsuperscript{25}

The social planner solves the same problem as Section 3 except that the utility function is now of general form. Again, the optimal tax rate is indeterminate because tax rates do not affect the social welfare (footnote 23). By the same procedure as in Section 3, the optimality condition for the environmental regulation is

$$-C'(\alpha^o) = -(1 + \beta) \frac{U_e}{U_x} \frac{1}{\theta_m=1}. \quad (15)$$

\textsuperscript{24}Aside from endogenous regulation, the current paper is different from Ogawa and Wildasin (2009) in that the current paper considers large jurisdictions which take into account terms of trade effects. Thus, some readers may wonder whether this modification to Ogawa and Wildasin (2009) changes their efficiency result. However, Eichner and Runkel (2012) show that when jurisdictions are symmetric, the efficiency result of Ogawa and Wildasin (2009) remains to hold even if jurisdictions are large. This is because terms of trade effects vanish at the symmetric equilibrium. Since this paper considers a symmetric situation, the assumption of large jurisdictions itself does not break the efficiency result.

\textsuperscript{25}Another possible justification for this social welfare function is the following. The social planner may tax capital so that $\rho = 0$ and redistribute the revenue equally because a typical social welfare function can be maximized by equalizing utilities of citizens. Given the complete redistribution of the returns to capital, everyone has the average amount of capital. Thus, the social planner effectively chooses regulation levels to maximize the utility of the voter with $\theta_m = 1$. Note, however, that the equilibrium tax rates affect the social welfare of the economy in this case. Thus, unlike the case in which the social welfare function is the sum of the utility of voters with $\theta_m = 1$, just comparing the equilibrium regulation levels and that chosen by the social planner is insufficient to obtain the full-fledged welfare implication. For this purpose, it is necessary to specify the distribution of capital endowments as well as the form of the social welfare function.
Regulation cost is paid by numeraire while the environmental damage directly affects the utility. The term \( \frac{U_x}{U_e} \) adjusts the relative importance of monetary cost and environmental damage. Other than this, it is the same as (9).

Following the same procedure, I solve the game in which the utility function is replaced with \( U(x_i, e_i) \). The first order conditions for \( t_i \) are, for \( i = 1, 2 \) and \( j \neq i \),

\[
\frac{\partial k_i}{\partial t_i} [-k_i f''(k_i) + t_i + \frac{U_e}{U_x} (\alpha_i - \beta \alpha_j)] + \frac{\partial \rho}{\partial t_i} \theta D k_i + k_i = 0. \tag{16}
\]

Using (16), the first order condition for \( \alpha_i \) is, at the symmetric equilibrium,

\[
-C'(\alpha^c) = -\frac{U_e}{U_x} |_{\theta_u=\theta^D} \tag{17}
\]

Note that these expressions reduce to those in case of linear utility by setting \( U_x = 1 \) and \( U_e = -1 \).

These expressions imply that the basic properties of the model do not crucially rely on the linearity of the utility function. When the utility is non-linear, the marginal rate of substitution \( \frac{U_x}{U_e} \) matters. Intuitively, the marginal rate of substitution \( \frac{U_x}{U_e} \) enters this expression for the following reason. Suppose a jurisdiction marginally strengthens its environmental regulation. It directly reduces the environmental damage, whose benefit is evaluated at \( U_e \). On the other hand, raising taxes increases the revenue and this benefit is evaluated at \( U_x \). Both reduction in \( \alpha \) and increase in \( \alpha \) indirectly affects utility by changes capital allocation and the net return to capital. However, using the similar intuition as in Proposition 1, the ratio of the indirect benefit by strengthening regulation and by increasing the tax rate should be \( \frac{U_x}{U_e} \). Thus, \( \frac{U_x}{U_e} \), the relative importance of the consumption \( x \) and the environmental damage \( e \), affects the regulation level at the equilibrium.

The marginal rate of substitution depends on the endowment of capital. (15) and (17) coincide if and only if \( \theta^D = 1 \). Indeed, comparing (15) and (17) under \( \theta^D = 1 \), it can be confirmed that Proposition 1, my main result, does not hinge on the linearity: it holds under general utility function.

**Proposition 2**
When \( \theta^D = 1 \), Proposition 1 holds even when the utility function of individuals is of general form.

Now, let me suppose \( \theta^D \neq 1 \). I assume \( \frac{U_x}{U_e} \) is strictly decreasing in \( x \) and \( e \). I show that in regular cases, the equilibrium regulation is stricter as \( \theta^D \) gets larger.

**Proposition 3**
When \( \theta^D \) is sufficiently close to 1 and the utility function is of general form, the equilibrium regulation level becomes stricter (looser) as \( \theta^D \) becomes larger (smaller).

**Proof:** See the Appendix.

Intuitively, Proposition 3 holds because a more capital-rich decisive voter enjoys larger amount of consumption. The relative importance of environmental quality is increased, which makes her

\[26\text{This assumption corresponds to the diminishing marginal rate of substitution in the standard consumer theory and it is a natural requirement. To see this, note that it holds if we assume that the utility is separable, the marginal utility of consumption is decreasing in } x \ (U_{xx} < 0), \text{ and the marginal environmental damage is increasing in } e \ (U_{ee} < 0).\]
choose stricter regulation.\footnote{This result may seem surprising because the net return to capital $\rho$ matters a lot for the capital-rich decisive voter and $\rho$ is increased by loosening regulation. What (17) shows is that, at the equilibrium, only the marginal rate of substitution matters. The marginal rate of substitution is ensured to be increasing in $\theta^D$ as long as the consumption is increasing in $\theta^D$ at the equilibrium, which is a natural situation. Note also that, while the analysis in Section 5 and Subsection 6.1 is specific to the inequality in capital, Proposition 3 does not necessarily rely on the inequality of capital endowment. As long as the amount of consumption is unequal for some reason (e.g., inequality in the endowment of the numeraire goods), the same conclusion would be obtained.} Note that, as implied by the proof of Proposition 3, the assumption that $\theta^D$ is sufficiently close to 1 is just a strong sufficient condition to ensure the regular behavior of the model. Thus, in practice, Proposition 3 would be valid in many circumstances.

Given that the equilibrium regulation level is efficient when $\beta = 0$ and $\theta^D = 1$ (Proposition 2), Proposition 3 implies that the equilibrium regulation is inefficiently strict (loose) if the decisive voter has slightly more (less) capital than the average. I summarize this result as the following corollary.

**Corollary 1**

Suppose that there is no transboundary pollution ($\pi = 0$). When $\theta^D$ is slightly larger (smaller) than one and the utility function is of general form, the equilibrium regulation level is too strict (too loose).

Corollary 1 shows the possibility of “race to the top” in the sense that the regulation is too strict compared with the optimum. However, Corollary 1 does not take into account the presence of transboundary pollution. When $\beta > 0$, the optimal regulation level is stricter since both $(1 + \beta)$ and $-\frac{U_\pi}{U_e}$ are increasing in $\beta$ at the equilibrium. Even in this case, when $\theta^D > 1$ and $\beta$ is not very large, the equilibrium regulation can be too strict due to the same mechanism as Corollary 1.

To illustrate this point, I conduct a numerical analysis incorporating transboundary pollution. I specify $U(x_i, e_i) = \log x_i - \frac{1}{2} e_i^2$ and $f(k) = mk - \frac{n}{2} k^2$, where $m, n > 0$ are parameters.\footnote{Quadratic production function is widely used in the literature (e.g., Bucovetsky, 1991; Peralta and Ypersele, 2005; Ogawa and Susa, 2017). Strictly speaking, the quadratic production function does not satisfy the technical conditions $f'(0) = \infty$, $f'(\infty) = 0$. These assumptions are made to ensure the positive marginal productivity. In the numerical analysis, I choose parameter values which guarantee the positive productivity. The assumption $F''' > 0$ is also not satisfied, but the analyses in Section 4 do not rely on this assumption.} The regulation cost is given by $C(\alpha) = \frac{1}{\alpha}$. I set $\bar{k} = 1$, $m = 5$, and $n = 0.5$. I consider three values of $\theta^D = 0.7, 1, 1.3$. I also consider two situations in which transboundary pollution is not severe ($\pi = 0$) and is severe ($\pi = 0.8$).

The results are reported in Table 2. Regulation levels become stricter as the decisive voter has

### Table 2: Equilibrium regulation levels in the numerical examples. All numbers are rounded off to two decimal places.

<table>
<thead>
<tr>
<th>Optimum ($\alpha^o$)</th>
<th>$\beta = 0.1$</th>
<th>$\beta = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^D = 0.7$</td>
<td>0.64</td>
<td>0.61</td>
</tr>
<tr>
<td>$\theta^D = 1$</td>
<td>0.66</td>
<td>0.57</td>
</tr>
<tr>
<td>$\theta^D = 1.3$</td>
<td>0.63</td>
<td>0.54</td>
</tr>
</tbody>
</table>
more capital, which is consistent with Proposition 3. When transboundary pollution is moderate ($\beta = 0.1$) and the voter is capital-rich ($\theta^D = 1.3$), the equilibrium regulation is too strict. In other cases, it is too loose. Thus, the result of Proposition 3 and Corollary 1 are confirmed even in the presence of transboundary pollution.

The analysis in this section shows that uncoordinated environmental regulation is inefficient in the presence of transboundary pollution. It implies that the efficiency result of Ogawa and Wildasin (2009) relies on the assumption of exogenous environmental regulation. Relaxation of it generally leads to an inefficient outcome. Although “race to the bottom” is likely to occur in many cases, Proposition 3 and Corollary 1 show the possibility that uncoordinated regulation may become too strict (“race to the top”). Thus, whether regulation should be stricter or looser has to be carefully considered before trying to reach an agreement. However, in any case, efforts to achieve an international environmental agreement are necessary to attain an optimal outcome.

Finally, I make two remarks. First remark is on the value of $D$ in the real world. The value of $\theta^D$ can be inferred from works which try to understand whose preference governments reflect. The validity of the median voter theorem is tested by a lot of works (e.g., Congleton, 2004). If the median voter theorem is valid, $\theta^D < 1$ would be plausible given the significant inequality in capital. On the other hand, Gilens and Page (2014) empirically show that economic elites and organized groups representing business interests have much more influence on US policymaking than average citizens and mass-based interest groups. Their result suggests $\theta^D > 1$ may be more plausible, although more empirical works are needed to reach a final conclusion.\footnote{$\theta^D > 1$ is also likely if capital-poor people are not allowed to vote. An example is the early days of the US, where only property-owning white men had voting rights. Historically, non-rich people were also often excluded from voting systems in other countries (e.g., UK and Japan). I thank a referee for drawing my attention to this point.}

Second, I briefly discuss what is going to happen if voters have heterogeneous preferences. So far, all voters share the same utility function, but they may differ in preferences. In particular, it is often argued that preferences for environmental quality become stronger as a person becomes richer.\footnote{However, empirical evidence on this issue is somewhat mixed. See Fairbrother (2013) for more discussion.} If this is the case, the equilibrium regulation when the decisive voter is capital-rich would tend to become too strict since she strongly prefers better environmental quality compared with many other citizens. Therefore, the implication of Proposition 3 and Corollary 1 would be strengthened.

5 Regulation-Tax Model

In this section, I alter the timing assumption and consider the case in which tax rates are chosen after regulation levels are determined. As discussed at the end of Section 2, this timing would be highly relevant because, in a realistic setting, a government may have to commit to certain regulation level for relatively a long time. For example, it would be the case if the new regulation requires firms to introduce new facilities or invent new technologies. In this section, I modify the timing assumption
in the following way. In the first stage, jurisdictions independently and simultaneously set regulation levels. In the second stage, they independently and simultaneously set tax rates. In the third stage, capital moves and utilities are realized. Other details are the same as the baseline model in Section 2. I call this model as “regulation-tax model.”

I solve the game by backward induction. What happens in the third stage is explained in Section 2: hence, I first analyze the second stage. In the second stage, jurisdiction \( i (=1, 2) \) chooses its capital tax rate to maximize (6) subject to (4) and (5). Jurisdiction \( i (=1, 2) \) regards as given the regulation levels \( \alpha_1, \alpha_2, \) and the opponent’s tax rate \( t_j \) \((j \neq i)\). The first order condition is

\[
\frac{\partial U_i}{\partial t_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial t_i} + \frac{\partial U_i}{\partial p} \frac{\partial p}{\partial t_i} = 0.
\]

Calculating this, I have

\[
k_i f''(2\overline{k} - k_i) - \theta^D \overline{k} f''(2\overline{k} - k_i) - \alpha_i + \beta \alpha_j + t_i = 0
\]

(18)

for \( i = 1, 2 \) and \( j \neq i \).

In the first stage, jurisdiction \( i (=1, 2) \) independently and simultaneously chooses its environmental regulation level \( \alpha_i \). Note that jurisdiction \( i \)'s regulation level \( \alpha_i \) affects the resulting capital tax rates determined by (18). The first order condition for the optimal \( \alpha_i \) is

\[
\frac{dU_i}{d\alpha_i} = \frac{\partial U_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial p} \frac{\partial p}{\partial \alpha_i} + \frac{\partial U_i}{\partial t_i} \frac{\partial t_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial \alpha_j} \frac{\partial \alpha_j}{\partial \alpha_i} + \frac{\partial U_i}{\partial \alpha_j} \frac{\partial \alpha_j}{\partial \alpha_i} + \frac{\partial U_i}{\partial \alpha_j} \frac{\partial \alpha_j}{\partial \alpha_i} = 0
\]

(19)

where the second equality follows from the first order condition of the second stage: \( \frac{\partial U_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial p} \frac{\partial p}{\partial \alpha_i} = 0 \). Since each jurisdiction optimally sets its capital tax rate in the second stage, the effect through \( t_i \) is 0 (the envelope theorem). However, the effect through \( t_j \) is not necessarily 0. I refer to \( \frac{\partial U_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial p} \frac{\partial p}{\partial \alpha_i} \) as the “non-strategic benefit” because this effect does not involve any strategic consideration. This effect solely comes from the change in \( \alpha_i \) and adjustments in the capital market. On the other hand, I call \( \frac{\partial U_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial \alpha_i} \) as the “strategic benefit” because this term represents the benefit from affecting the opponent’s behavior in the second stage. Note that the strategic benefit has been ignored in most of previous literature such as Oates and Schwab (1988). I focus on the symmetric equilibrium with \( \alpha_1 = \alpha_2 = \alpha^c \) and \( t_1 = t_2 = t^c \).

I first calculate the non-strategic benefit. Using (5) and (18) and noting that \( \alpha_1 = \alpha_2 = \alpha^c \) and \( k_1 = k_2 = \overline{k} \) at the symmetric equilibrium,

\[
\frac{\partial U_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial p} \frac{\partial p}{\partial \alpha_i} = -\overline{k} - \overline{k} C''(\alpha^c).
\]

(20)

Note that all regulation cost is internalized by the government regardless of \( \theta^D \).

Now, I calculate the strategic-benefit. Using (4), (5), and (18) and then imposing the symmetric condition, I obtain

\[
(\frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial p} \frac{\partial p}{\partial \alpha_i}) \frac{\partial t_i}{\partial \alpha_i} = (1 - \theta^D)\overline{k} \frac{\partial t_i}{\partial \alpha_i}.
\]

(21)
It is evident from (21) that the strategic benefit is zero if $\theta^D = 1$. When $\theta^D = 1$, the assumption on capital ownership essentially reduces to the standard one made in Ogawa and Wildasin (2009). In this case, the strategic benefit is always zero at the equilibrium regardless of $\frac{\partial t_j}{\partial \alpha_i}$, the effect of environmental regulation on the tax rate of the other jurisdiction. The strategic benefit can be decomposed into two effects: the effect through $k_i$ and the effect through the net return to capital $\rho$. Suppose the tax rate of jurisdiction $j$ slightly rises. This induces capital outflow into jurisdiction $i \neq j$ and the decrease in the net return to capital. Acquiring more capital is beneficial to the individual with $\theta^D = 1$ because it increases the wage rate and the tax base. However, the decrease in the net return to capital is harmful because they invest their capital endowment. When $\theta^D = 1$, these two effects offset each other, resulting in the zero strategic benefit.

Using (20) and (21), the first order condition (19) at the symmetric equilibrium can be rewritten as

$$\frac{dU_i}{d\alpha_i} = -k - kC'(\alpha^c) + (1 - \theta^D)k\frac{\partial t_j}{\partial \alpha_i} = 0.$$  \hfill (22)

When $\theta^D = 1$, it is evident from (22) that the regulation is inefficiently loose if $\beta > 0$, while it is efficient if $\beta = 0$. The following proposition shows a much more strong result: this efficiency characterization is valid for very wide range of $\theta^D$.

**Proposition 4**

Suppose that tax rates are chosen after regulation levels are chosen.

(i) Suppose that $\theta^D > 0$ and $\theta^D$ is not too large.\(^{31}\) Then, the uncoordinated environmental regulation under intergovernmental competition $\alpha^r$ is inefficiently loose ($\alpha^c > \alpha^o$) as long as there is transboundary pollution ($\beta \in (0, 1]$). It is efficient ($\alpha^c = \alpha^o$) if there is no transboundary pollution ($\beta = 0$).

(ii) Suppose $\theta^D = 0$. Then, the equilibrium is efficient for all $\beta \geq 0$ (i.e., regardless of the degree of transboundary pollution).

**Proof:** See the Appendix.

Proposition 4(i) confirms the robustness of my main result (Proposition 1) under the alternative timing assumption. That is, the regulation is inefficiently loose (efficient) in the presence (absence) of transboundary pollution. When $\theta^D$ is not equal to one, strategic benefit is nonzero. Thus, voters use regulation levels to manipulate the tax rates in the second stage, which seems to make the characterization of Proposition 1 invalid. However, Proposition 4 indicates that the strategic benefit works so that qualitative predictions of Proposition 1 are preserved. When $\theta^D < 1$ so that the decisive voter is capital-poor, she wants to increase the other jurisdiction’s tax rate because it induces capital inflow and increase the wage rate and the tax base. On the other hand, when

\(^{31}\)This assumption a sufficient condition to eliminate irregular behavior of the model, which I explicitly state in the Proof. Importantly, $\theta^D$ can always be larger than one.
$\theta^D > 1$, she wants to decrease it because it increases the net return to capital. The sign of $\frac{\partial t_j}{\partial \sigma_i}$ is, in many plausible circumstances, negative.\textsuperscript{32} This implies that capital-poor voters want to choose strict regulation. When the transboundary pollution is not severe, the optimal regulation level is also not very severe. However, the absolute value of $\frac{\partial t_j}{\partial \sigma_i}$ tends to be small when the transboundary pollution is not very severe.\textsuperscript{33} Since small $\frac{\partial t_j}{\partial \sigma_i}$ in absolute value diminishes the incentive for the capital-poor voter to strengthen regulation, the regulation level does not become very strict when the optimal regulation level is loose due to small $\beta$. Thus, as long as $\theta^D > 0$, the qualitative prediction of Proposition 1 is preserved.

On the other hand, Proposition 4(ii) shows the possibility that the efficiency result of Ogawa and Wildasin (2009) may be restored due to the strategic benefit. If $\theta^D = 0$ so that the decisive voter has no capital, the equilibrium is efficient even in the presence of transboundary pollution. In this case, the strategic benefit is sufficiently important for the decisive voter. When $\theta^D$ is exactly equal to zero, this benefit just internalizes the transboundary pollution. Proposition 4(ii) extends the efficiency result of Ogawa and Wildasin (2009) to a situation in which regulation levels are also endogenously chosen as in Oates and Schwab (1988). It would be serve as a benchmark in evaluating the applicability of the efficiency result of Ogawa and Wildasin (2009) when both tax rates and regulation levels are endogenously chosen.

However, it is important to note that Proposition 4(ii) is only a limit result with respect to the value of $\theta^D$. Thus, the inefficiency result of Proposition 1 is essentially preserved in the regulation-tax model. I make two further remarks on Proposition 4(ii). First, the same result is obtained if capital is assumed to be owned by absentee owners. In the literature of capital tax competition, the results sometimes crucially depends on the ownership of capital (Ogawa, 2013). Second, Proposition 4(ii) relies on the strategic benefit, which arises because each jurisdiction’s policymaking has a non-negligible effect on other jurisdictions’ policies. Thus, Proposition 4(ii) does not hold when jurisdictions are infinitesimally small or they ignore such effect.

Although the qualitative implication of Proposition 1 remains generally intact, quantitative predictions can be affected due to the presence of the strategic benefit. In particular, the results of

\textsuperscript{32} The explicit expression of $\frac{\partial t_j}{\partial \sigma_i}$ is given in the Appendix. It is positive only if $-C' > 1$. Intuitively, when jurisdiction $i$ loosen its regulation, it steals capital from jurisdiction $j$. The capital outflow decreases the wage and jurisdiction $j$ can decrease the tax rate to recover the wage rate by attracting more capital. Moreover, the shrinkage in the tax base also reduces the incentive to set a high tax rate. $-C' > 1$ ensures that these effects are sufficiently strong. It can be shown that $-C' > 1$ at the equilibrium if $\theta^D < 1$. This result ensures that at the equilibrium, the strategic benefit makes capital-poor voter want to strengthen regulation.

\textsuperscript{33} To intuitively understand the statement, suppose that jurisdiction $i$ loosens its regulation, which increases the pollution in jurisdiction $i$. When there is no transboundary pollution, it has no effect on the utility of jurisdiction $j$. However, if there is transboundary pollution, it increases the total environmental damage in jurisdiction $j$. To reduce pollution in jurisdiction $j$, jurisdiction $j$ can set lower tax rates to move capital from jurisdiction $i$. As a result, the absolute value of $\frac{\partial t_j}{\partial \sigma_i}$ is larger when transboundary pollution is severer. Note that, strictly speaking, this statement assumes that when transboundary pollution becomes severer, $\frac{\partial t_j}{\partial \sigma_i}$ is not evaluated at the much looser regulation level. The violation of this assumption cannot occur except when $\theta^D$ is sufficiently larger than one (see equation 34 and 35).
Proposition 4 imply that the strategic benefit may act against the driving force of Proposition 3. The specification of the utility function and the timing assumption matter in obtaining a quantitative welfare implication as realistic as possible. Thus, they may deserve careful empirical investigation, although it is beyond the scope of this paper.

6 Discussions

In this section, I further discuss the robustness of the my main result (Proposition 1).

6.1 Tax-Regulation Model

Now, I adopt another alternative timing assumption that regulation levels are independently and simultaneously chosen after tax rates are chosen independently and simultaneously. This assumption may be plausible if, for example, governments cannot easily manipulate tax rates, possibly because of political considerations. The model is the same as that in Section 5 except that the timing is reversed. I call this model “Tax-Regulation Model.” Specifically, in the first stage, jurisdictions independently and simultaneously set tax rates. In the second stage, they independently and simultaneously set regulation levels. In this case, tax rates are chosen taking into account their effect on the choice of regulation levels. This strategic consideration may change the tax rate, which in turn affect the choice of regulation levels. I demonstrate the characterization in Proposition 1 remains valid even in this case.

The FOC in the second stage is, for \( i = 1, 2 \) and \( j \neq i \),

\[-k_i[f''(k_i) + f''(k_j)] - C'(\alpha_i)[k_i f''(k_i) - t_i + \alpha_i + \theta P_k f''(k_j) - \beta \alpha_j] = 0. \tag{23}\]

The FOC in the first stage is, after utilizing the envelope theorem,

\[
\frac{dU_i}{d t_i} = \frac{\partial U_i}{\partial t_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial t_i} + \frac{\partial U_i}{\partial p} \frac{\partial p}{\partial t_i} + \left( \frac{\partial U_i}{\partial \alpha_j} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial \alpha_j} + \frac{\partial U_i}{\partial p} \frac{\partial p}{\partial \alpha_j} \right) \frac{\partial \alpha_j}{\partial t_i} = 0 \tag{24}\]

for \( i = 1, 2 \) and \( j \neq i \). Using terminologies in Section 5, the last term corresponds to ‘strategic benefit’ while the remaining terms are ‘non-strategic benefit.’ Changing tax rates may affect the regulation level of jurisdiction \( j \) in the second stage, which affects the utility of jurisdiction \( i \).

Using (23), at the symmetric equilibrium \( (t_i = t_j = t^c, \alpha_i = \alpha_j = \alpha^c) \), (24) can be rewritten as

\[
(-C'(\alpha^c) - 1)[P_k f''(\bar{k})] - t^c + (1 - \beta) \alpha^c + \theta P_k f''(\bar{k}) + [2(-1 - \beta)P_k f''(\bar{k}) - 2C'(\alpha^c)\theta P_k f''(\bar{k})] \frac{\partial \alpha_j}{\partial t_i} = 0, \tag{25}\]

where the first term of the LHS is the non-strategic benefit and the second term is the strategic benefit.

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34When tax rates are chosen after regulation levels are chosen, the presence of the strategic benefit makes capital-poor (capital-rich) voters choose stricter (looser) regulation in many cases. This is a counter-force against Proposition 3. Thus, although the inefficiency result remains valid, the size of the efficiency loss may be greatly affected.
(23) and (25) determine $t^c$ and $\alpha^c$. These two equations are so complicated that complete analytical characterization of solutions seems infeasible. However, by focusing on the case of $\theta^D = 1$, I can show the following proposition.

**Proposition 5**

Suppose that regulation levels are chosen after tax rates are chosen. Suppose also $\theta^D = 1$. Then, the equilibrium regulation level is inefficient if $\beta > 0$, while it is efficient if $\beta = 0$.

**Proof**: See the Appendix.

Proposition 5 indicates that the result similar to Proposition 1 holds even under this alternative timing assumption, although the direction of inefficiency is now ambiguous.

Combined with Proposition 4, Proposition 5 suggests that timing assumption does not alter the main qualitative result that uncoordinated regulation is inefficient in the presence of transboundary pollution. The result that it is efficient in the absence of both transboundary pollution and politically biased preference is also quite robust.

### 6.2 Multiple Jurisdictions

My analysis has been confined to the case of two jurisdictions. This case has attracted much attention in the literature (e.g., Bucovetsky, 1991), but the case in which there are many jurisdictions is also important (e.g., Zodrow and Mieszkowski, 1986). In this section, I consider the case in which $n$ is larger than two while keeping the assumptions of simultaneous choice and linear utility.

There are $n$ symmetric jurisdictions with population $L = 1$ and the initial endowment of capital $\bar{k}$, implying that the total amount of capital is $n\bar{k}$. Except for the number of jurisdictions, the model is the same as one in Section 2.

Since there are $n$ jurisdictions, the environmental damage (2) is modified as

$$e_i = \alpha_i k_i + \beta \sum_{j \neq i} \alpha_j k_j,$$

which is again a natural extention of that in Ogawa and Wildasin (2009). The size of transboundary pollution is increasing in $n$ since the number of affected countries is increased.

Following the same procedure as in Section 3, the first-best regulation is characterized by

$$-C'(\alpha^o) = 1 + (n - 1)\beta,$$

where the LHS of (27) represents the social marginal cost of additional regulation and the RHS represents the social marginal benefit.

Evaluated at the symmetric distribution of capital ($k_i = \bar{k}$ for all $i$), the reaction of capital is modified as follows:

$$\frac{\partial k_i}{\partial t_i} = \frac{n - 1}{n f''(\bar{k})} < 0, \quad \frac{\partial k_i}{\partial t_j} = \frac{-1}{n f''(\bar{k})} > 0, \quad \frac{\partial \rho}{\partial t_i} = \frac{-1}{n} < 0$$

(28)
and
\[
\frac{\partial k_i}{\partial \alpha_i} = \frac{(n-1)C'(\alpha_i)}{nf''(k)} > 0, \quad \frac{\partial k_i}{\partial \alpha_j} = \frac{-C'(\alpha_j)}{nf''(k)} < 0, \quad \frac{\partial p}{\partial \alpha_i} = \frac{-C'(\alpha_i)}{n} > 0. \tag{29}
\]

The FOC for \( t_i(i = 1, \ldots, n) \) evaluated at the symmetric equilibrium is given by
\[
(n-1)(-kf''(k) + t_i - \alpha_i) + \sum_{j \neq i}^n \beta \alpha_j - \theta^Dk f''(k) + nf''(k)k = 0. \tag{30}
\]

The FOC for the regulation level is rewritten as, at the symmetric equilibrium,
\[
\frac{\partial U_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial k_i} \frac{\partial k_i}{\partial \alpha_i} + \frac{\partial U_i}{\partial \rho} \frac{\partial \rho}{\partial \alpha_i} + \sum_{j \neq i} \frac{\partial U_i}{\partial k_j} \frac{\partial k_j}{\partial \alpha_i} = -k + \frac{\partial k_i}{\partial \alpha_i} (-kf''(k) + t_i - \alpha_i) + \frac{\partial \rho}{\partial \alpha_i} \theta^Dk - \beta \sum_{j \neq i} \alpha_j \frac{\partial k_j}{\partial \alpha_i} = -k - C'(\alpha^c)k. \tag{31}
\]

The second equality comes from (30).

(31) is rewritten as
\[
-C'(\alpha^c) = 1 \tag{32}
\]
where \( j \neq i \). Comparing (27) and (32), I obtain the following proposition:

**Proposition 6**

Even when there are more than two jurisdictions, the equilibrium regulation level is characterized in the same way as Proposition 1.

Proposition 6 confirms the robustness of my main result that uncoordinated regulation is inefficiently loose in the presence of transboundary pollution. When \( n \) is increased, the size of the externality increases. However, each jurisdiction does not take this into account. Thus, the degree of the inefficiency would be exacerbated. It confirms the robustness of Proposition 1.

In concluding this subsection, I make two remarks on how my other results are modified when \( n \) is larger than two. First, when \( n \) is larger, ‘race to the top’ is less likely to occur because the gap between the optimal regulation (\( \alpha^o \)) and the equilibrium regulation (\( \alpha^c \)) becomes larger as the size of the transboundary pollution increases. Still, if the utility function is non-linear and the decisive voter has sufficiently large amount of capital, the regulation may become too strict due to the mechanism explained in Proposition 3 and Corollary 1.

Second, as I note in Section 5, strategic benefit vanishes when \( n \) is large. This is because each jurisdiction is so small that it has no impact on policymaking of the other jurisdictions. Thus, if \( n \) is infinitely large, the strategic benefit no longer matters in the regulation-tax model and the efficiency of the regulation is characterized in the same way as Proposition 1. It implies that the characterizations in Proposition 4(i) are quite robust to the changes in \( n \) but Proposition 4(ii) is not because it relies on the magnitude of the strategic benefit.
7 Conclusion

Under capital tax competition, Ogawa and Wildasin (2009) show that uncoordinated policymaking leads to an efficient outcome even in the presence of transboundary pollution. I show, however, that their efficiency result crucially depends on the implicit assumption that the levels of environmental regulation are given exogenously. If regulation levels are endogenized, the presence of transboundary pollution makes uncoordinated regulation too loose (“race to the bottom”). In other words, intergovernmental competition for capital does not internalize transboundary pollution when jurisdictions compete not only in taxes but also in regulation levels. Thus, as opposed to the implication of Ogawa and Wildasin (2009), efforts to achieve an international coordination are needed to restore efficiency. Moreover, the main qualitative result, intergovernmental competition via both taxes and regulation levels does not restore efficiency in the presence of transboundary pollution, is quite robust to various modifications to the model. Notably, this result holds even when regulation levels may affect equilibrium tax rates.

I consider the possibility that governments maximize the utility of the decisive voter, rather than the average utility of residents as in Ogawa and Wildasin (2009). Recognizing the fact that a government may act according to the interest of the median voter or some political parties, a number of works on tax competition introduce political economic factors to see how they affect the results (e.g., Borck, 2003; Sato, 2003; Lai, 2014; Ogawa and Susa, 2017). In this case, although uncoordinated regulation generally turns out to be inefficient as before, I point out the possibility that uncoordinated regulation becomes inefficiently strict (“race to the top”). This result holds mainly because at the equilibrium, voters with large capital endowment value the environmental quality more than many other voters. Although the inefficiency result is highly robust with respect to the endowment of capital, I also show that in the limit case where the decisive voter has exactly no capital and tax rates are chosen after regulation levels, the efficiency result of Ogawa and Wildasin (2009) may be restored.

My results clearly point out that the presence of transboundary pollution generally leads to an inefficient outcome under intergovernmental competition for capital. However, the analysis in this paper does not incorporate some important elements in environmental issues. Most notably, both jurisdictions are assumed to be symmetric. Eichner and Runkel (2012) note that in an asymmetric situation, an additional terms of trade effect plays a role, which complicates the analysis. Although uncoordinated regulation levels differ from those in the symmetric case, jurisdictions’ asymmetry does not alter the result that uncoordinated environmental policymaking generally leads to an inefficient outcome. Even so, characterizing welfare properties of uncoordinated policymaking in an asymmetric case would be interesting. Bucovetsky (1991) considers capital tax competition between two jurisdictions which differ in population size, and shows that the terms of trade effect may benefit the small jurisdiction. Therefore, it could be conjectured that a small jurisdiction is more likely
to benefit even when pollution and environmental regulation are incorporated, although the full analysis of this case remains for future research.

**Appendix: Mathematical Proofs**

**Proof of Proposition 3**

Consider the hypothetical game in which regulation levels of both jurisdictions are fixed at an arbitrary value \( \alpha \) and that the tax rates are determined in the tax competition (i.e., determined by (16)). I prove the following two lemmas on the symmetric equilibrium of this game.

**Lemma 1**

If \( \theta^D \) is sufficiently close to 1, at the symmetric equilibrium, \( -\frac{U_e}{U_x} \) must be increasing in \( \alpha \).

**Proof:** Let \( x \) and \( e \) be the amount of consumption and the environmental damage at the equilibrium. These are functions of \( \alpha \) since the equilibrium values depend on the exogenously given \( \alpha \). Then, noting that I evaluate the expression at the symmetric equilibrium,

\[
\frac{\partial}{\partial \alpha} \left( -\frac{U_e}{U_x} \right) = -\left[ \left( \frac{\partial (U_e)}{\partial x} \right) \frac{\partial x}{\partial \alpha} + \left( \frac{\partial (U_e)}{\partial e} \right) \frac{\partial e}{\partial \alpha} \right] > 0.
\]

(33)

because \( \frac{\partial x}{\partial \alpha}, \frac{\partial e}{\partial \alpha} > 0 \) and \( \frac{\partial (U_e)}{\partial x}, \frac{\partial (U_e)}{\partial e} < 0 \). To see why \( \frac{\partial x}{\partial \alpha} > 0 \), note that the increase in \( \alpha \) increases the net return to capital \( \rho \) by saving the regulation cost. The change in \( \alpha \) may induce the change in the equilibrium tax rates determined by (16), but it does not affect the consumption of the decisive voter as long as \( \theta^D = 1 \) since, at the equilibrium, tax increase changes the net return to capital \( (\rho k) \) and the amount of transfer from the government \( (t_i k) \) by the same amount. Thus, \( \frac{\partial x}{\partial \alpha} > 0 \) when \( \theta^D \) is sufficiently close to 1 due to continuity. \( \frac{\partial e}{\partial \alpha} > 0 \) holds because the amount of capital has to be \( k \) at the symmetric equilibrium. Thus, \(-\frac{U_e}{U_x}\) is increasing in \( \alpha \). ■

**Lemma 2**

Fix any \( \alpha \). If \( \theta^D \) is sufficiently close to 1, at the symmetric equilibrium, \(-\frac{U_e}{U_x}\) is larger when \( \theta^D \) is larger.

**Proof:** Take any \( \theta_a^D \) and \( \theta_b^D \) with \( \theta_a^D < \theta_b^D \). Given the common regulation level \( \alpha \), the equilibrium condition in the second stage (16) must hold. At the symmetric equilibrium, the increase in \( \theta^D \) unambiguously increases the consumption \( x \) through increasing the capital endowment. The increase in \( \theta^D \) also induces changes in the equilibrium tax rates determined by (16), but, at the equilibrium, it has no effect on the amount of consumption when \( \theta^D = 1 \) because at the equilibrium, tax increase changes the net return to capital \( (\rho k) \) and the amount of transfer from the government \( (t_i k) \) by the same amount. Thus, when \( \theta_a^D \) and \( \theta_b^D \) are sufficiently close to 1, the increase in \( \theta^D \) unambiguously increases the consumption.

Since the amount of capital is independent of \( \theta^D \) at the symmetric equilibrium and the regulation level is exogenously given, the environmental damage \( e \) is also independent of \( \theta^D \). Since the
consumption is larger when $\theta^D$ is larger while the environmental damage is constant, it follows that under the common regulation level $\alpha$, $-\frac{U_x}{U_z}$ is larger when $\theta^D$ is larger. ■

**Proof of the Proposition:** Take any $\theta^D_a$ and $\theta^D_b$ with $\theta^D_a < \theta^D_b$. I assume both $\theta^D_a$ and $\theta^D_b$ are sufficiently close to 1 that both Lemma 1 and Lemma 2 can be applied. Let $\alpha^c|_{\theta^D=\theta^D_b}$ be the equilibrium regulation level when $\theta^D = \theta^D_b$. Suppose that the regulation level of both jurisdictions are exogenously set at $\alpha = \alpha^c|_{\theta^D=\theta^D_b}$ and the tax rates are determined to satisfy (16). Noting that $\alpha^c|_{\theta^D=\theta^D_b}$ satisfies (17) when $\theta^D = \theta^D_b$, from Lemma 2, the RHS of (17) is larger than the LHS at $\alpha = \alpha^c|_{\theta^D=\theta^D_b}$ when $\theta^D = \theta^D_b$. Given Lemma 1 and that the LHS is strictly decreasing in $\alpha$, to jointly satisfy (16) and (17) when $\theta^D = \theta^D_b$, the equilibrium regulation level $\alpha^c|_{\theta^D=\theta^D_b}$ must be smaller than $\alpha^c|_{\theta^D=\theta^D_a}$. In words, the equilibrium regulation becomes stricter (looser) when the decisive voter has more (less) capital. ■

**Proof of Proposition 4**

Applying the implicit function theorem to (18) at the symmetric equilibrium,

$$\frac{\partial t_j}{\partial \alpha_i} = -\beta(1 + M) + M + MC'(\alpha^c)$$

where

$$M \equiv \frac{f''(\tilde{k}) - (1 - \theta^D)f''(\tilde{k})\tilde{k}}{2f''(\tilde{k})}.$$ 

$M > 0$ if and only if $\theta^D < 1 - \frac{f''(\tilde{k})}{k f''(\tilde{k})} \equiv \theta^*$. Since $f'' < 0$ and $f'' > 0$, $\theta^* > 1$. Herewith, I assume $\theta^D < \theta^*$ so that $M > 0$.

The first order condition (22) is now rewritten as

$$-C'(\alpha^c) = \frac{(2M + 1) - (1 - \theta^D)[-\beta(1 + M) + M]}{2M + 1 - (1 - \theta^D)M}.$$ 

The denominator of (35) is always positive. The numerator of (35) becomes negative if $\beta > \frac{M}{M+1}$ and $\theta^D > \frac{-\beta(1 + M + 1)}{-\beta(1 + M) + M}$. When $\beta > \frac{M}{M+1}$, I assume $\theta^D < \frac{-\beta(1 + M + 1)}{-\beta(1 + M) + M}$ to ensure that the numerator of (35) is also always positive.$^{36}$

$$1 + \beta - \frac{2M + 1 - (1 - \theta^D)[-\beta(1 + M) + M]}{2M + 1 - (1 - \theta^D)M} \text{ is}$$

$$\frac{\beta \theta^D(2M + 1)}{2M + 1 - (1 - \theta^D)M} \geq 0,$$ 

and it holds with equality if and only if $\beta = 0$ or $\theta^D = 0$. Thus, from (9) and (35), I obtain the results. ■

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$^{35}$Here, I mean the equilibrium regulation level of the game in which both taxes and regulation levels are endogenously chosen. That is, $\alpha^c|_{\theta^D=\theta^D_b}$ is the regulation level when the (symmetric) tax rate and the regulation level are determined by (16) and (17) when $\theta^D = \theta^D_b$.

$^{36}$In the range $\frac{M}{M+1} < \beta \leq 1$, $\frac{-\beta(M+1)}{-\beta(1+M)+M}$ takes the minimum value $2M + 2$ when $\beta = 1$. Thus, this restriction allows for $\theta^D$ larger than 2. It implies that the restriction is not very severe.
Proof of Proposition 5

First, I consider the case of $\beta > 0$. Suppose $-C'(\alpha^c) = 1 + \beta$ so that $\alpha^c$ is efficient. From (25), this implies that $t_i = 2\overline{k}f''(\overline{k}) + (1 - \beta)\alpha^c$. Substituting $-C'(\alpha^c) = 1 + \beta$ and $t_i = 2\overline{k}f''(\overline{k}) + (1 - \beta)\alpha^c$ into (23), I get $2\overline{k}f''(\overline{k}) = 0$, which is a contradiction. Thus, $-C'(\alpha^c) \neq 1 + \beta$ so that the equilibrium regulation is inefficient.

Next, suppose $\beta = 0$. Then, it is straightforward to verify that $\alpha^c$ defined by $-C'(\alpha^c) = 1$ and $t_i = \alpha^c$ solve (23) and (25). Moreover, if $-C'(\alpha^c) \neq 1$, there is no tax rate which makes both (23) and (25) hold at the equilibrium. To see this, note that (25) implies $t_i = 2\overline{k}f''(\overline{k}) + (1 - \beta)\alpha^c + 2\overline{k}f''(\overline{k}) \frac{\partial \alpha^c}{\partial t_i}$, but it does not generally solve (23). Thus, the equilibrium regulation level is efficient. ■

References


